Discriminants and Electrostatic Models for Orthogonal Polynomials

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Abstract: Stieltjes and Hilbert derived closed form expressions for discriminants of Jacobi polynomials. Stieltjes studied the electrostatics equilibrium problem of n-unit charged particles restricted to (-1,1) under the external field of charges (a+1)/2 and (b+1)/2 at 1 and -1. The potential is a logarithmic potential. Stieljes showed that the equilibrium position of the particle is at the zeros of the Jacobi polynomial with parameters a and b. We discuss the recent developments on this problem, its extension to general orthogonal polynomials and the role discriminants play in the solution of the problem. This includes deriving first order raising and lowering operators and second order differential equtations for general orthogonal polynomials. Expressions for discriminants of general orthogonal polynomials are also given. Finally generalization of discriminants are given and some applications will be briefly mentioned.